The equation of continuity for a homogeneous electron beam is

$$\frac{1}{\sigma_b}\frac{dI_b}{dz} + j\omega\rho_b = 0. \tag{5}$$

From (1), (2), (3) and (5), we obtain

$$\frac{dI_b}{dz} = \left(\frac{j\omega I_0}{2u_0 V_0}\right)V_b, \tag{6}$$

where, under the small signal conditions, the ac beam voltage is

$$V_b = \frac{u_0 v_b}{\eta}. \tag{7}$$

The equation of motion for the lossless plasma is

$$j\omega v_p = \eta E, \tag{8}$$

from which, by using (2) and (4), we obtain

$$I_p = \frac{\sigma_p}{\sigma_b}\cdot\frac{\omega_p^2}{\omega^2}\cdot\frac{1}{\left(1 - \dfrac{\omega_p^2}{\omega^2}\right)}\cdot I_b. \tag{9}$$

By proper manipulation of (1), (2), (7) and (8), the following equation is obtained:

$$\frac{dV_b}{dz} = \frac{j}{\omega\epsilon_0\sigma_b}\,\frac{1}{\left(1 - \dfrac{\omega_p^2}{\omega^2}\right)}\cdot I_b. \tag{10}$$

It is noted that (6) is the same equation derived by Bloom and Peter for the case of a modulated beam.[1] Eq. (10) is, however, modified by the factor

$$\left(\frac{1}{1 - \dfrac{\omega_p^2}{\omega^2}}\right),$$

which takes the presence of plasma into account. It may be shown that these two simultaneous differential equations [(6) and (10)] describing the beam-plasma interaction are formally identical with those of the transmission line loaded with lumped resonant circuits, as shown in Fig. 1. We have, from Fig. 1,

$$\frac{dV}{dz} = jXI; \qquad \frac{dI}{dz} = jBV. \tag{11}$$

There is, therefore, a complete correspondence if line voltage V and line current I correspond to the ac beam voltage V_b and the ac beam current I_b respectively.

The essential feature to be considered here is that the series impedance is capacitive for $\omega > \omega_p$ and inductive for $\omega < \omega_p$. This behavior is very similar to a continuous multicavity klystron or easitron where the resonators are tuned at a frequency such that they present a lossless negative susceptance to the electron beam. It is known in this case of an inductive circuit admittance that there exist increasing waves in the system.[2] For $\omega < \omega_p$, therefore, the beam-plasma system would lead to growing and decaying waves, as has been verified experimentally by several authors.[3,4]

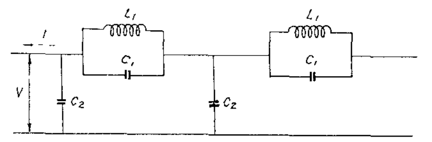

Fig. 1—Line analog of an electron beam in a lossless plasma.

$$C_2 = \epsilon_0\sigma_b\left(\frac{2\pi}{\lambda_p}\right)^2 \text{farads}/m \qquad C_1 = \epsilon_0\sigma_b \text{ farads}/m$$

$$L_1 = \frac{1}{\eta\sigma_b\rho_{0p}} \text{ henrys}/m \qquad \omega_p^2 = \frac{1}{L_1C_1}.$$

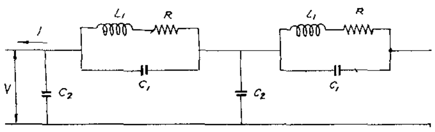

Fig. 2—Line analog of an electron beam in a plasma, with collisions taken into account.

$$R = \nu L_1 = \frac{\nu}{\eta\sigma_b\rho_{0p}} \text{ ohms}/m$$

where ν = plasma collision frequency.

Losses in the plasma from collision effects are readily included in the analog line by the introduction of a resistive component in the line series impedance, as shown in Fig. 2.

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[2] J. R. Pierce, "Waves in electron streams and circuits," *Bell Sys. Tech. J.*, vol. 30, pp. 626–651, July, 1951.

[3] G. D. Boyd, L. M. Field and R. W. Gould, "Excitation of plasma oscillations and growing plasma waves, *Phys. Rev.*, vol. 109, pp. 1393 1394; February, 1958.

[4] G. F. Freire, "Interaction effects between a plasma and a velocity-modulated electron beam," Microwave Lab., Stanford University, Stanford Calif.; Tech. Rept. No. 890; February, 1962.

Correction to "A Method for Measuring the Directivity of Directional Couplers"

B. C. Yates of the Radio Standards Laboratory, National Bureau of Standards, Boulder, Colo., has brought to the attention of the authors of the above paper[1] errors in some of the equations for error limits. The authors have investigated the difficulty and derived correct equations. If one uses the erroneous expressions, error limits are obtained which are much too conservative.

The equations in question are numbered (18) and (19). The correct expressions are

$$20\log_{10}\frac{1 - |\Gamma_{2i}|^2}{1 - \left|\dfrac{\Gamma_{2i}}{K}\right|} \geq \epsilon_{II,1} \geq 20\log_{10}\frac{1 - |\Gamma_{2i}|^2}{1 + \left|\dfrac{\Gamma_{2i}}{K}\right|}, \tag{18}$$

and

$$\epsilon_{II,2} = 20\log_{10}\frac{1 - |\Gamma_{2i}|^2}{1 - \left|\dfrac{\Gamma_{2i}}{K}\right|^2}. \tag{19}$$

It should be noted that the approximation $y/2 \approx (\Gamma_{2i}/K)$ is no longer needed in the derivations of (18) and (19), and that (19) can represent a correction rather than an error limit if $|\Gamma_{2i}|$ and $|K|$ are known. In order for (18) and (19) to hold,

$$|\Gamma_{2i}| < \frac{1 - \dfrac{1}{|K|}}{1 + \dfrac{1}{|K|}},$$

but the values of $|\Gamma_{2i}|$ and $|K|$ normally encountered will be found to satisfy this inequality.

The graphs of Fig. 5, which were based upon (18) and (19), are no longer correct. However, it was found that, for procedure 1, a sufficiently accurate answer can be obtained by dividing the decibel error limits by 3.

Fig. 5 does not give the correct results for procedure 2. However, it was found that for $|K^2| > 10$, (19) can be considered as insensitive to directivity, and equal to 20 $\log_{10}(1 - |\Gamma_{2i}|^2)$.

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Manuscript received January 10, 1964.

[1] G. E. Schafer and R. W. Beatty, IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-6, pp. 419–422; October, 1958.

Multifrequency Microwave Generation Using a Large Capacitance Tunnel Diode

Fundamental, simultaneous oscillation at two discrete microwave frequencies has been experimentally verified using an inexpensive tunnel diode. The diode possesses relatively high junction capacitance of 90 pf (see Fig. 1).

The diode, whose cost is below three dollars, was a 1N3718 and was mounted in an impedance transformation waveguide mount,[1] as illustrated in Fig. 1. In this case the effects of package and junction capacitance are not reduced. The diode package is performing as a cavity resonator, resonating in the X band. The diode was found to oscil-

Manuscript received November 12, 1963; revised January 27, 1964.

[1] C. C. Hoffins and K. Ishii, "Microwave tunnel-diode operation beyond cutoff frequency," PROC. IEEE (*Correspondence*), vol. 51, pp. 370–371; February, 1963.

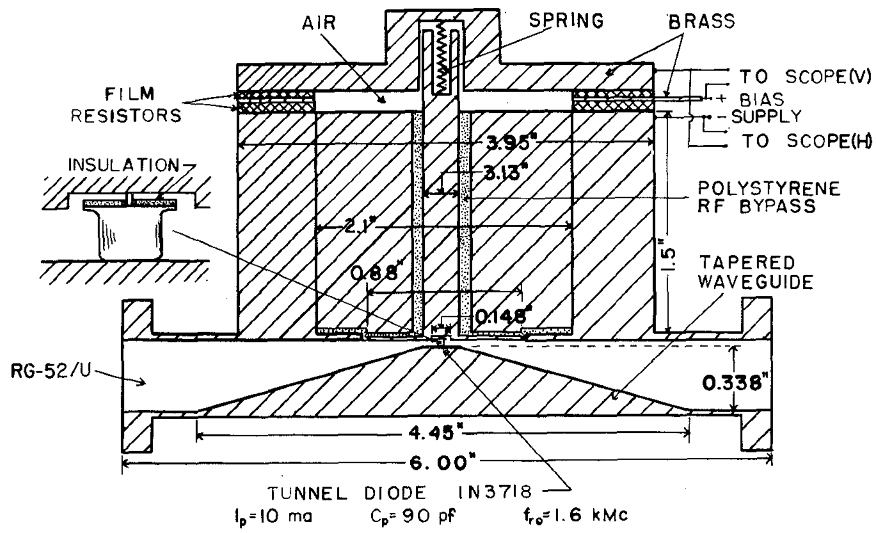

Fig. 1—1N3718 Tunnel Diode mounted in impedance transformation waveguide mount.

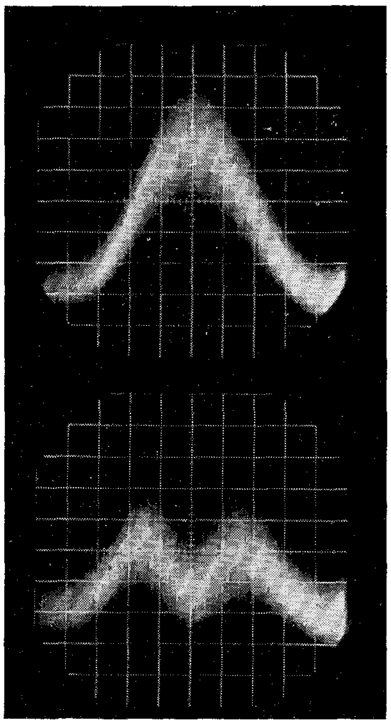

Fig. 2—Detected output of 1N3718 with a dc bias of 215 mv and at a frequency of 11,480 Mc.

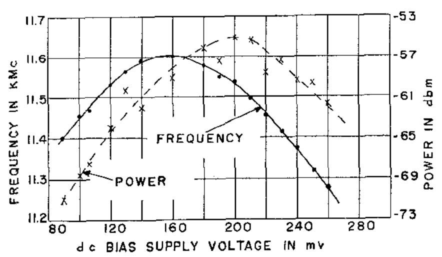

Fig. 3—Effect of dc bias voltage on the *X*-band oscillation and output power.

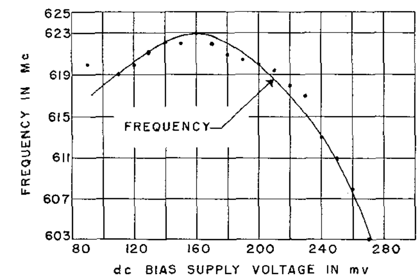

Fig. 4—Effect of dc voltage on the UHF oscillation.

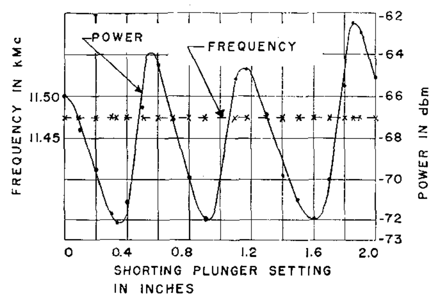

Fig. 5—Effect of the shorting plunger on the 11,480 Mc frequency of oscillation and the output power.

late at two discrete fundamental frequencies,[2] 618 Mc and 11,480 Mc, at a bias voltage of 215 mv. These frequencies were detected using a Polarad RW-T receiver and a SA-84WA spectrum analyzer. The detected and H.P.-X532B wavemeter-dipped oscillation at 11,480 Mc is shown in Fig. 2. Plots of frequency vs bias voltage for the observed oscillations are presented in Figs. 3 and 4. The 11.5-kMc oscillation was verified with the shorting plunger (see Fig. 5). From this experiment the average waveguide half-wavelength was determined to be 0.625 inches, which corresponds to a frequency of 11,493 Mc.

The first nine harmonics of the 618-Mc fundamental were detected using a coaxial probe, and are presented in column A of Table I. From this data the average fundamental was determined to be 614 Mc. Using this average fundamental, the harmonics were calculated and are presented in column B. The observed 11,480-Mc oscillation does not fall within the range of the theoretical harmonics of the 614-Mc fundamental. Even when the equipment detection error is accounted for, the error analysis still indicates that the 11,480-Mc oscillation is not harmonically related to 614 Mc. Also, an analysis of frequency deviation with bias voltage indicates that 11,480 Mc does not fall within the harmonic range of 614 Mc.

Thus, microwaves have been generated with a low-frequency, large capacitance, inexpensive tunnel diode.

The authors wish to thank J. A. Stefancin, S. Krupnik, Jr., J. E. Billo, E. Starr, G. H. Folland, D. Simonich and C. C. Hoffins for their assistance in this investigation.

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[2] K. Ishii and C. C. Hoffins, "Oscillation frequency of microwave tunnel diodes," Proc. IEEE (*Correspondence*), vol. 51, pp. 485–486; March, 1963.

TABLE I
Observed and Theoretical Frequencies of Oscillation

Harmonics	A Experimental Frequencies	B Theoretical Frequencies
Fundamental	618	614
2	1240	1228
3	—*	1842
4	2434	2456
5	3050	3070
6	3652	3684
7	4265	4298
8	4863	4912
9	5495	5526
10	—*	6140
11	—*	6754
12	—*	7368
13	—*	7982
14	—*	8596
15	—*	9210
16	—*	9824
17	—*	10438
18	—*	11052
19	—*	11666
20	—*	12280

Bias = 215 mv.
* Not experimentally detected.

An Improved Mode-Selecting Coupler

A mode-selecting coupler which couples two orthogonally oriented rectangular waveguides into circular or square waveguide for simultaneous orthogonal transmission has been previously reported by Ohm.[1] It has been found that by adding an additional septum to the coupler configuration the bandwidth can be substantially improved.

As shown in Fig. 1, rectangular guide is coupled to square guide through a reso-

Manuscript received January 29, 1964. This work was supported by RCA, Defense Electronics Products, Surfcom Division, Cambridge, Ohio.

[1] E. A. Ohm, "A broad-band microwave circulator," IRE Trans. on Microwave Theory and Techniques, vol. MTT-4, pp. 210–217; October, 1956.